Moral Hazard in Strategic Decision Making*

Martin C. Byford†

Abstract

This paper develops a theory of managerial incentives based on the manager's role as strategic decision-maker within the firm. Career concerns give rise to preferences over risk, creating an incentive for the manger to manipulate the firm's risk at the expense of (expected) profits. The resultant moral hazard can be ameliorated by an incentive contract. However, contracting is complicated by the failure of the Monotone Likelihood Ratio Condition to hold.

A solution is proposed in which a Non-Decreasing Wage Constraint is incorporated into the contracting problem. This solution overcomes the practical problems created by a non-monotone likelihood ratio at the expense of discarding some of the information present in the firm's profits. The implications of the non-decreasing wage constraint are illustrated in a simple example in which the second-best contract is option-like, with a 'strike-price' that is strictly less than the firm's expected profit and decreasing in both the firm's risk and the magnitude of the moral hazard problem.

This Version: June 22, 2015

J.E.L. Classification: D82, D83, L14.
Keywords: Principal-agent, moral hazard, managerial incentives, career concerns, stock-options.

* I am grateful to Joshua Gans, Michael Harris, Stephen King and Simon Anderson for comments and advice on earlier versions of this paper. This research was made possible by the a grant from the *Research Program on the Economics of Knowledge Contribution and Distribution*.

† School of Economics, Finance and Marketing, RMIT University. Building 80 Level 11, 445 Swanston Street, Melbourne VIC 3000, Australia. Email: martin.byford@rmit.edu.au.

1 Introduction

The structure of managerial incentives have been studied extensively in the moral hazard literature. The usual analysis focuses on the problem of motivating a manager to exert effort (early references include Mirrlees, 1976, 1999; Holmstrom, 1979). In contrast, the manager's role as the strategic decision-maker in the firm has received relatively little attention.

Decisions taken by a firm's manager determine the firm's behaviour in the various strategic environments in which it operates. For example, the manager prices the firm's products, selects the size of production runs, and chooses which markets to contest. If the manager operates at arm's length from the firm's shareholders, these strategic decisions will be susceptible to moral hazard.

One might reason that moral hazard is not a factor in strategic decision-making as the manager and shareholders share a common incentive to maximise profits. After all, a manager's career prospects tend to be linked to the performance of their firm. However, as Holmstrom and Ricart i Costa (1986) (see also Holmstrom, 1999) demonstrate, the manager's career concerns can also give rise to preferences over risk, potentially distorting the manager's choice of strategy.[1]

Indeed, Holmstrom and Ricart i Costa (1986) argue that a firm's profits are likely more susceptible to the moral hazard associated with strategic decision-making than the moral hazard associated with managerial effort. On the one hand, managers are not typically effort averse. Indeed, the career path to senior management seems designed to identify individuals who experience a relatively low disutility of effort. On the other hand, the manager's strategic decisions can be the difference between substantial profits and bankruptcy.

This paper develops a theory of managerial incentives based on the manager's role as strategic decision-maker within the firm. The paper adapts the career concerns model of Dewatripont, Jewitt and Tirole (1999a,b, henceforth DJT) to show how a manager's future

[1] In their model a risk-averse manager faces a binary choice, to commission or reject a risky project. Holmstrom and Ricart i Costa (1986) show that the manager may reject a project with positive expected profits where failure would provide a negative signal of the manager's talent, harming the manager's future earnings.

employment prospects create incentives that distort the decisions she takes on behalf of her current employer.[2] In the model, which is outlined in section 2, a risk-averse manager lives and works for two periods. Each period the manager seeks employment in the labour market. The manager's employment prospects depend on the market's beliefs concerning her talent. These beliefs are updated following the manager's first-period performance, creating career concerns that influence her first-period behaviour.

The model is distinguished from the usual moral hazard analysis by the nature of the manager's task within the firm. The manager is the agent responsible for selecting and implementing the firm's strategy. For example, she might be required to determine the firm's behaviour in a standard model of oligopoly such as Hotelling or Cournot competition.[3] The choice set is assumed to be a connected subset of $\mathbb{R}$ with three characteristics that distinguish the choice of strategy from the problem of choosing an effort level:

1. *The manager is indifferent between the strategies in the choice set:* For the purposes of this paper it is assumed that the manager has no innate preference over the set of strategies that are available to the firm. This is a strong assumption. However, it ensures that the only factors influencing the manager's actions are her career concerns and any incentives that are incorporated into her contract.

2. *The (expected) profit maximising strategy is in the interior of the choice set:* A feature of many models of oligopoly is that the profit maximising strategy lies in the interior of the choice set. For example, the optimal capacity for a factory is unlikely to be the maximum capacity that a firm can construct. An interior optimum has implications for the contracting problem as it precludes the Monotone Likelihood Ratio Condition.

3. *There exists a trade-off between reducing (or increasing) a firm's risk and maximising profits in the neighbourhood of the profit maximising strategy:* In choosing a strategy for a

[2] In contrast, DJT (1999a,b) investigate the role that career concerns can play in motivating effort in the absence of incentive contracts.

[3] Models of advertising expenditure, R&D, location choice or quality choice are also consistent with the analysis of this paper.

firm, a manager typically has a degree of control over the amount of risk associated with the firm's profits. If the manager's career concerns create preferences over risk then she has an incentive to select a strategy other than the profit maximising strategy.

In section 3 it is shown that the first of these characteristics, combined with the structure of the manager's career, implies the first-best outcome is achievable in the second period. If the manager is offered a fixed wage contract then, insulated from the firm's risk, she will weakly prefer to implement the profit maximising strategy. In contrast, the manager's first-period behaviour will be influenced by the risk preferences derived from her career concerns. If the manager is either risk-averse or risk-seeking with respect to firm profits, she has an incentive to distort the firm's strategy away from the profit maximising value in order to manipulate the firm's risk.

The problem of utilising an incentive contract to ameliorate the first-period moral hazard is considered in section 4. The firm's profits carry information about the strategy implemented by the manager. However, it is shown that the Monotone Likelihood Ratio Condition cannot hold at the second-best strategy, implying that the second-best contract will decrease in the firm's profit over some range and potentially invalidating the first-order approach (see for example Mirrlees, 1999; Rogerson, 1985; Jewitt, 1988).

There are, of course, a number of practical problems associated with a wage that falls as the firm's profit rises. Such a wage rewards failure and may create an incentive for the manager to sabotage the firm or implement a perverse strategy. A straightforward solution is to impose an additional constraint on the contracting problem, requiring that the manager's wage be non-decreasing in firm profits. This constraint resolves all the problems associated with a non-monotone likelihood ratio. However, it also prevents the contract from exploiting the information present in a given level of profit if the likelihood ratio implies that the manager's wage should be decreasing in profit at that point.

The model provides an explanation for incentive contracts, such as option contracts, that disregard the information present in a range of outcomes. To illustrate this phenomenon further, a simple and intuitive example is developed in section 5. In the example extreme profits, both high and low, are indicative of risk taking, while moderate profits imply a risk-averse choice of strategy. If the manager's career concerns leave her risk-averse with respect to the firm's profit then the second-best contract is option-like. The 'strike price' for the contract is strictly less than the firm's expected profit implying that the manager will be paid a 'bonus' if profits meet expectations. Moreover, the strike price is decreasing in both the firm's risk and the magnitude of the moral hazard problem.[4]

2 The Model

The career concerns model in this paper is adapted from DJT (1999a,b). A manager (the agent) lives and works for two periods. Each period the manager seeks employment in the labour market. The employers in the labour market are the owners of firms (the principals) who require the specialised skills of a manager to oversee the operation of their firms.

Contracts in the labour market apply for a single period. Neither the manager, nor the owners of a firm, can make commitments in first period that dictate employment, wages or behaviour in the second period.[5] Because the manager must reenter the labour market following the first period, her second-period career concerns create incentives that may influence her first-period behaviour.

2.1 Timing

The timing of the model, illustrated in figure 1, is as follows: The first-period begins with the labour market. Prospective employers approach the manager with take-it-or-leave-it offers of a contract. Contracts may specify an incentive structure. The manager reviews the contracts on offer and accepts her preferred contract.

[4] Within the moral hazard literature, option contracts have previously been explained as optimal responses to loss aversion (de Meza and Webb, 2007) and bounds on payments (Jewitt, Kadan and Swinkels, 2008). In each case the problem under consideration was motivating the manager to exert effort.

[5] This assumption does not preclude the manager from being employed at the same firm in both periods. Rather, it requires the owners of a firm to match the manager's best outside offer following her first-period performance if they are to retain the manager's services. Moreover, it allows for the manager to loose her position if first-period performance indicates that the owners would be better off replacing the manager.

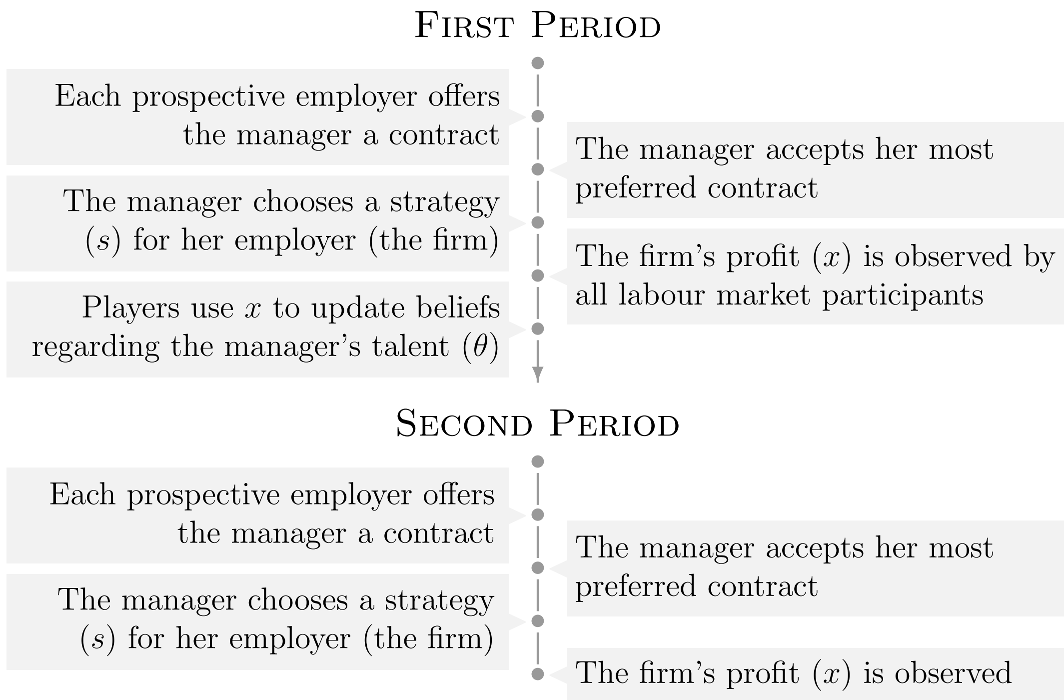

Figure 1: The Manager's Career

Following the labour market, the manager commences work at her employer's firm. The manager's task is to select a strategy for the firm and oversee its implementation. The manager's actions are hidden from both the firm's owners and the wider labour market.

The firm's profits are realised at the end of the period. Profits are stochastic and contingent on both the strategy implemented by the manager and the manager's talent. All labour market participants, including the owners of the firm, observe the firm's profits and use the firm's profits to update their beliefs regarding the manager's talent.

The manager comes out of contract between periods, reentering the labour market at the start of the second period. The second period proceeds with the same timing as the first. The only difference between periods is that there is no need for the market to update beliefs regarding the manager's talent at the conclusion of the second period as this is the final period of the manager's career.

2.2 The Manager

The manager is risk averse and motivated only by the wages she receives over her career. Specifically, it is assumed that the manager's lifetime utility takes the form,

$$U(w, v) = u(w) + \delta u(v), \tag{1}$$

where w and v are the manager's first- and second-period wages respectively, and $\delta \in (0, 1)$ is the manager's discount factor. The manager's risk aversion implies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Implicit in (1) are the assumptions that the manager can neither borrow against future earnings, nor save for future consumption. These assumptions are not necessary, but do simplify the analysis considerably.

2.3 The Manager's Role in the Firm

In delegating control of their firm to the manager, the owners grant the manager considerable autonomy to direct the operations of the firm. Formally, the manager's task is to select the firm's strategy s from the set of all viable strategies S. It is assumed that the manager's activities within the firm are sufficiently hidden from the firm's owners, and the labour market as a whole, that s cannot be observed or contracted upon.

In order to simplify the analysis S is assumed to be a connected subset of $\mathbb{R}$. For example, $s \in S$ might represent the capacity of a new factory, or the quality of a product being developed by the firm. The construction of S means that within the model the manager enjoys a fine degree of control over the firm's strategy.

A key assumption of this paper, clearly illustrated in (1), is that the manager's utility is independent of the actions she takes while managing a firm, except insofar as these actions impact on the manager's first- and second-period wages. In other words, it is assumed that the manager does not have innate preferences over the set of strategies that are available to the firm.

The firm's profit during the manager's tenure $x \in X$ is stochastic and depends both on the firm's strategy s and the talent of the manager $\theta \in \Theta$, where X and Θ are likewise connected subsets of $\mathbb{R}$. The distribution of x is written $F(x|s,\theta)$, while $f(x|s,\theta)$ is the corresponding density function.[6] It is assumed that f has full support and is twice continuously differentiable in all of its arguments. The firm's profit is publicly observable and verifiable.

Knowledge of the manager's talent is imperfect. At the start of the first period all labour market participants share the manager's prior belief that θ is drawn from a distribution with density function $g(\theta)$.[7] It is assumed that g has full support and is twice continuously differentiable. Given $g(\theta)$, the first-period marginal density function is,

$$\hat{f}(x|s) = \int_{\Theta} g(\theta) f(x|s,\theta) d\theta, \tag{2}$$

for all $x \in X$ and $s \in S$. The corresponding marginal distribution is denoted $\hat{F}(x|s)$.

2.4 Strategy, Talent and Profit

The following assumptions describe the relationship between s, θ and x, and highlight the difference between the model in the present paper and the usual moral hazard analysis.

Assumption 1: For any given prior $g(\theta)$, it is assumed that the marginal distribution $\hat{F}(x|s)$ satisfies the following conditions:

(a) $\hat{F}_{ss}(x|s) > 0$ for all $x \in X$ and $s \in S$.[8]

(b) Define,

$$\tilde{s} \equiv \underset{s \in S}{\operatorname{argmax}} \int_X x \hat{f}(x|s) dx. \tag{3}$$

There exist $s, s' \in S$ such that $s < \tilde{s} < s'$.

(c) The variance of x is strictly increasing in s.[9]

[6] Each firm in the labour market has its own, possibly unique, profit distribution $F(\cdot)$. Moreover, the set of viable strategies S, and the range of each firm's potential profits x, may also vary from firm to firm. As the identity of the manager's firm is never ambiguous within the model, and in the interests of notational simplicity, distribution functions are not indexed by firm in this paper.

[7] The assumption of a common prior is standard in the career concerns literature as it removes adverse selection from the model. See for example Holmstrom (1999), Harris and Holmstrom (1982), Holmstrom and Ricart i Costa (1986) and DJT (1999a,b).

[8] Throughout the paper subscripts denote partial derivatives, hence $\hat{F}_{ss}(x|s) = \partial^2 \hat{F}(x|s)/\partial s^2$.

[9] Formally, $\frac{\partial}{\partial s}\left[\int_X x^2 \hat{f}(x|s)dx - \left(\int_X x\hat{f}(x|s)dx\right)^2\right] > 0$.

The three conditions in assumption 1 have straightforward interpretations. Assumption 1a is the standard Convexity of the Distribution Function Condition (CDFC) (see for example Grossman and Hart, 1983; Rogerson, 1985; Jewitt, 1988). Where the CDFC holds the expected value of any increasing function $h(x)$,

$$E\big[h(x)|s\big] = \int_X h(x)\hat{f}(x|s)dx,$$

is strictly concave in s. It follows that the strategy $\tilde{s}$, that maximises the firm's expected profits, is unique.

Assumption 1b states that $\tilde{s}$ lies in the interior of S. Given that the manager's task is to select a strategy for the firm, this assumption implies that the strategic problem facing the firm has an interior solution; a common assumption in industrial organisation. Assumption 1b also implies that $\hat{f}(x|s)$ does *not* display the Monotone Likelihood Ratio Condition (MLRC).[10]

Finally, assumption 1c states that the manager's choice of strategy also has consequences for the risk associated with the firm's profit. Specifically, that the variance of x is strictly increasing in s. It follows that a manager with preferences over risk faces a tradeoff between maximising the expected profits of the firm, and manipulating the risk associated with these profits, in the neighbourhood of $\tilde{s}$.

The next assumption describes the role that the manager's talent plays in determining the firm's profits.

Assumption 2: For any given $s \in S$ and $\theta \in \Theta$ the likelihood ratio $f_\theta(x|s,\theta)/f(x|s,\theta)$ is strictly increasing in x.

Assumption 2 has two implications for the model. The first concerns the updating of beliefs following the revelation of the firm's first-period profits. Define s^* as the strategy that solves the manager's first-period optimisation problem. In other words, the strategy that market participants will assume that the manager selected. The posterior beliefs regarding

[10]If the likelihood ratio $\hat{f}_s(x|s)/\hat{f}(x|s)$ were increasing in x for all $s \in S$ then $E[x|s]$ would likewise be increasing in s, but this is not the case as $\tilde{s}$ lies in the interior of S.

the manager's talent are thus,

$$\bar{g}(\theta|x) \equiv g(\theta)\frac{f(x|s^*,\theta)}{\hat{f}(x|s^*)}, \tag{4}$$

for all $\theta \in \Theta$ and $x \in X$. Milgrom (1981) has shown that if $x'' > x'$ then assumption 2 implies that the posterior $\bar{g}(\theta|x'')$ dominates $\bar{g}(\theta|x')$ in the sense of strict first-order stochastic dominance.

The second implication of this assumption concerns the value of a manager's talent to the owners of a firm. A likelihood ratio that is monotone in θ implies that for any given strategy s, a firm's expected profit,

$$E[x|s,\theta] = \int_X xf(x|s,\theta)dx,$$

is strictly increasing in θ.

3 Moral Hazard

The owners of firms are assumed to be risk neutral. Within the model owners are concerned only with maximising the expected profits of their firms, net of the manager's wage. In contrast, the manager is risk averse with strictly concave utility. Given these risk preferences, first-best risk sharing requires the owners of a firm to offer the manager a contract with a fixed wage.

Moral hazard arises in the model when a fixed-wage contract results in a tension between the preferences of the manager and those of the firm's owners. In this section it is shown that no such tension exists in the second period, however career concerns can create divergent preferences over risk in the first period.

3.1 First-Best in the Second Period

The second period is the final period of the manager's (working) life and as such career concerns do not influence the manager's behaviour in this period. Because the manager has no innate preferences over the strategies available to a firm, it is possible to achieve a first-best outcome in the second period.

The second period begins with the labour market. The manager prefers fixed-wage contracts, over incentive contracts with the same expected wage, as a fixed-wage contract insulates the manager from risk. Prospective employers have no reason to offer the manager an incentive contract as the manager already weakly prefers to implement the strategy that maximises the firm's expected profits. This is the first-best outcome.

While the manger's second-period wage will not depend on the manager's second-period performance, the wages that prospective employers offer the manager in the second-period labour market will depend on the market's beliefs concerning the manager's talent. From (4) it is apparent that these beliefs depend, in turn, on the profit produced by the manager in the first period.

PROPOSITION 1: *The expected profit of the manager's second-period employer is strictly increasing in the profit produced by the manager in the first period.*

Proof. Suppose that the manager produces a profit of x' in the first period. The strategy that maximises the expected profit of the manager's second-period employer is,

$$\bar{s}(x') = \underset{s \in S}{\operatorname{argmax}} \int_X \int_\Theta x \bar{g}(\theta|x') f(x|s,\theta) d\theta dx,$$

where $\bar{g}(\theta|x')$ is defined in (4) and f is the density function of the manager's second-period employer. For any $x'' > x'$,

$$\begin{aligned}\int_X \int_\Theta x \bar{g}(\theta|x'') f\big(x|\bar{s}(x''),\theta\big) d\theta dx &\geq \int_X \int_\Theta x \bar{g}(\theta|x'') f\big(x|\bar{s}(x'),\theta\big) d\theta dx \\ &> \int_X \int_\Theta x \bar{g}(\theta|x') f\big(x|\bar{s}(x'),\theta\big) d\theta dx,\end{aligned}$$

where the first inequality follows from the construction of $\bar{s}(\cdot)$ and the second inequality arises because $\bar{g}(\theta|x'')$ dominates $\bar{g}(\theta|x')$ in the sense of strict first-order stochastic dominance. □

Proposition 1 states that in the second-period labour market, the value of the manager to the owners of a firm is strictly increasing in the profit produced by the manager in the first period. Where the second-period labour market is sufficiently competitive, the following assumption follows naturally from proposition 1.

Assumption 3: The manager's second-period wage v is a function of the manager's first-period profit x. Moreover, $v'(x) > 0$ for all $x \in X$.

For any given x, the wage $v(x)$ represents the best offer that manager receives in the second-period labour market. Of course, the identity of the firm that makes the best offer may change with x.

3.2 Moral Hazard in the First Period

The dependence of the manager's second-period wage on first-period profit creates career concerns for the manager in the first period. The manager's career concerns are described by the function,

$$\psi(s) \equiv \delta \int_X u\big(v(x)\big)\hat{f}(x|s)dx, \tag{5}$$

which captures the impact of the manager's choice of strategy in the first period, on the manager's expected utility in the second period.[11] It is important to note that the continuity and convexity of $\hat{F}$ imply that $\psi(s)$ is twice continuously differentiable and that $\psi''(s) < 0$ for all $s \in S$.

At first glance it may appear as though the manager's career concerns align her incentives with those of the manager's first-period employer. After all, assumption 3 states that the manager's second-period wage is strictly increasing in the firm's profit. However, this reasoning neglects the fact that the manager's career concerns expose the manager to the risk associated with x.

Define $\tilde{\tilde{s}} \equiv \text{argmax}_{s \in S}\, \psi(s)$. In words, $\tilde{\tilde{s}}$ is the strategy that maximises the manager's second-period expected utility. This is the strategy the agent will select if she is on a fixed-wage contract in the first period. A tension exists between the preferences of the manager, and the owners of the firm, if $\tilde{s} \neq \tilde{\tilde{s}}$.

Proposition 2: *If $u\big(v(x)\big)$ is strictly concave (resp. convex) then $\tilde{s} > \tilde{\tilde{s}}$ (resp. $\tilde{s} < \tilde{\tilde{s}}$).*

Proof. The continuity of $\hat{f}$ and the construction of $\tilde{s}$ implies $\int_X x \hat{f}_s(x|\tilde{s})dx = 0$. It follows

[11] In (5), $\hat{f}$ represents the marginal density function of the manager's first period employer.

from assumption 1c that a marginal change in s, in the neighbourhood of $\tilde{s}$, alters the variance of x but not its expected value. From Jensen's inequality it follows that $\psi(s)$ is decreasing (resp. increasing) in s at $\tilde{s}$ when $u\big(v(x)\big)$ is strictly concave (resp. convex). The result then follows from the strict concavity of $\psi(s)$. □

Proposition 2 establishes sufficient conditions for moral hazard to arise in the first period. While the manager and the owners of the firm have common preferences over the firm's profits, if the manager's career concerns give rise to divergent preferences over risk, the manager and the owners of the firm will differ in their preferred strategy.

A manager for whom $u\big(v(x)\big)$ is concave will prefer a strategy $\tilde{\tilde{s}}$ that is less than $\tilde{s}$. Such a manager is willing to reduce expected profits in exchange for reduced risk. Conversely, a manager for whom $u\big(v(x)\big)$ is convex has risk-seeking preferences and prefers strategies that are higher than $\tilde{s}$.

The concavity of $u(\cdot)$ suggests that managers will tend to derive risk-averse preferences from their career concerns. However, if $v(\cdot)$ is very convex, as may be the case in an industry with superstars (see for example Rosen, 1981), the term $u\big(v(x)\big)$ may likewise be convex.

4 Contracting in the First Period

The first-period moral hazard problem can be ameliorated through the use of an incentive contract. By linking the manager's first period wage to the firm's profit, the owner's of the firm can provide the manager with incentives to select a strategy closer to $\tilde{s}$. The contracting problem is complicated by the failure of the MLRC to hold, however this problem can be overcome by imposing a monotonicity condition on the manager's wage.

4.1 The First-Order Approach

With the manager's career concerns described by the function $\psi(s)$, the principal-agent problem reduces to a familiar form. The first-order approach to the problem requires the owners of the firm to,

$$\max_{w(x)} \int_X \big(x - w(x)\big)\hat{f}(x|s^*)dx, \tag{6}$$

subject to the participation constraint,

$$\int_X u\big(w(x)\big)\hat{f}(x|s^*)dx + \psi(s^*) \geq \bar{U}, \tag{7}$$

and the incentive compatibility constraint,

$$\int_X u\big(w(x)\big)\hat{f}_s(x|s^*)dx + \psi'(s^*) = 0. \tag{8}$$

In this program $\bar{U}$ represents the expected lifetime utility of the manager from accepting her next best offer in the first-period labour market.

Let λ and μ be the multipliers for (7) and (8) respectively. Pointwise optimisation of the Lagrangian yields the familiar equations,[12]

$$\frac{1}{u'\big(w(x)\big)} = \lambda + \mu\frac{\hat{f}_s(x|s^*)}{\hat{f}(x|s^*)}, \tag{9}$$

and,

$$\mathrm{cov}\left[\frac{1}{u'\big(w(x)\big)}, u\big(w(x)\big)\right] = -\mu\psi'(s^*). \tag{10}$$

Given that $u''(\cdot) < 0$, if $\mu > 0$ then (9) implies $w(\cdot)$ is increasing (resp. decreasing) in x where $\hat{f}_s(x|s^*)/\hat{f}(x|s^*)$ is likewise increasing (resp. decreasing) in x. This relationship is reversed if $\mu < 0$.

The sign of μ is determined implicitly by (10). The LHS of (10) is unambiguously positive as $1/u'\big(w(x)\big)$ and $u\big(w(x)\big)$ covary in the same direction. It follows that μ must have the opposite sign to $\psi'(s^*)$. Intuitively, if the owners of the firm want to encourage the manager to adopt a more risky strategy, as is the case where $u\big(v(x)\big)$ is concave, then $\mu > 0$. The reverse is the case when the owners want to discourage risk taking. Thus the second-best contract rewards profits that are indicative of higher (resp. lower) values of s when the manager's career concerns create risk-averse (resp. risk-seeking) preferences.

Rogerson (1985) has shown that the first-order approach is not necessarily valid unless the manager's wage is monotone increasing in x.[13] If $w'(\cdot)$ is not positive for all $x \in X$ then

[12] See Jewitt (1988) for the derivation of these equations. Note that the negative sign on the term on the RHS of (10) arises because the $\psi(\cdot)$ function represents a benefit to the manager.

[13] The second condition identified by Rogerson (1985) is the CDFC. Assumption 1a ensures that the CDFC holds.

the manager's optimisation problem may not be concave in s and s^* may not represent a global maximum. From (9) it follows that a monotone wage requires that the likelihood ratio $\hat{f}_s(x|s^*)/\hat{f}(x|s^*)$ be either monotone increasing or decreasing in x. The following proposition establishes that no such monotonicity can hold at s^* as defined in (8).

PROPOSITION 3: *For any non-decreasing wage $w(x)$ the likelihood ratio $\hat{f}_s(x|s^*)/\hat{f}(x|s^*)$ is not monotone in x.*

Proof. By way of contradiction, suppose that $\hat{f}_s(x|s^*)/\hat{f}(x|s^*)$ is increasing in x; this is the local version of the MLRC. Using (5), the manager's incentive compatibility constraint (8) can be rewritten as,

$$\int_X \big(u(w(x)) + \delta u(v(x))\big)\hat{f}_s(x|s^*)dx = 0.$$

Note that the term $u(w(x)) + \delta u(v(x))$ is strictly increasing in x. But if $\hat{f}_s(x|s^*)/\hat{f}(x|s^*)$ is increasing in x then a marginal increase in s will strictly increase the expected value of any strictly increasing function of x, violating (8). A contradiction. An equivalent contradiction can be constructed for a monotone decreasing likelihood ratio using a marginal decrease in s. □

The second-best contract exploits the information present in the firm's profit. If the owners of the firm wish to encourage the manager to select higher values of s, they must reward outcomes that are indicative of such a choice; levels of x for which the likelihood ratio $\hat{f}_s(x|s^*)/\hat{f}(x|s^*)$ takes a relatively high value. Proposition 3 demonstrates that within this model higher profits do not always carry more favourable information. Combining this insight with (9) suggests that the second-best contract should reward failure, and punish success, over any range of profits where the product of μ and the likelihood ratio is decreasing in x.

4.2 Three Arguments Against Rewarding Failure

If a contract decreases in x over some range then there exist profits $x', x'' \in X$ with $x'' > x'$ such that $w(x') > w(x'')$. In words, the contract rewards the manager for producing the profit x', relative to x'', despite x' being the lower profit level. There are a number of practical reasons why the owners of the firm may be unwilling to write a contract that rewards failure in this way.

1. *A wage that is decreasing in firm profits over some range may encourage a perverse choice of strategy.* For the first-order approach to be valid, the strategy s^* that solves (8) must represent a global maximum to the manager's optimisation problem. This is the case if $w(x)$ is monotone increasing in x, as assumption 1a ensures that the manager's optimisation problem is strictly concave in s. If, however, $w(x)$ is decreasing in x over some range then there may exist $s' \in S$ such that $s' \neq s^*$ and,

$$\int_X u\big(w(x)\big)\hat{f}(x|s')dx + \psi(s') > \int_X u\big(w(x)\big)\hat{f}(x|s^*)dx + \psi(s^*).$$

The existence of the strategy s' invalidates the first-order approach and may have adverse effects on the firm's expected profits. Intuitively, if the contract places a sufficiently high reward on low profits, the manager has an incentive to implement a strategy the produces low profits with high probability.

2. *A wage that is decreasing in firm profits over some range may encourage ex-post sabotage.* The manager's position within the firm both allows the manager to take actions that are hidden from outsiders, and affords her superior information regarding the state of the firm. It is likely that the manager will learn of the firm's profit before it is revealed to the firm's owners and the market as a whole. Moreover, the manager may be able to take hidden actions that reduce the firm's profit before it is released to the public.

Where the manager's wage is non-decreasing in x the manager has no incentive to sabotage the firm's profits. However, where the manager's wage does decrease in x over some range, the manager may be able to increase her wage by generating additional costs or engage in other 'money burning' activities.

3. *There exists a cultural prohibition against rewarding failure/punishing success.* Many societies hold to the principal that success should be rewarded. Both the punishment of

success and rewarding of failure are regarded as perverse outcomes. Social attitudes are particularly relevant in the context of publicly traded firms. Shareholders are unlikely to condone executive compensation that rewards low profits and losses, relative to higher profit outcomes.

4.3 Unusable Information

If one accepts one or more of the preceding arguments, it is appropriate to impose a third constraint on the optimisation problem. The Non-Decreasing Wage Constraint (NDWC),

$$w'(x) \geq 0, \tag{11}$$

is not derived as a feature of the optimal contract, but rather imposed due to the practical considerations of contract design.

With (11) included in the program, the slope of the second-best contract is jointly determined by (9) and (11). Specifically, for any x at which,

$$\frac{\partial}{\partial x}\left(\mu\frac{\hat{f}_s(x|s^*)}{\hat{f}(x|s^*)}\right) \geq 0, \tag{12}$$

the NDWC is slack and slope of $w(x)$ is determined implicitly by (9). Conversely, for any x at which (12) does not hold, the NDWC is binding and $w'(x) = 0$.

Imposing (11) on the program has consequences for the way in which the information contained in the firm's profits is utilised in the second-best contract. Define the set of profits $X^- \subset X$ such that $x \in X^-$ if and only if (12) does not hold at x. Note that proposition 3 implies both $X^- \neq \varnothing$ and $X^- \neq X$. The information carried by profits in X^- cannot be utilised by the owners of the firm. To see this note that within X^- lower realisations of x imply a choice of strategy that was more favourable to the owners of the firm. The NDWC prevents the second-best contract from rewarding lower profits and hence the next-best solution is to ignore the information.

5 Stock Options

This section presents a straightforward and intuitive example for which the second-best contract shares a number of features in common with contracts that utilise stock options as an incentive device.

Suppose that the manager has been hired to oversee the construction of a new factory. The manager's task is to select a strategy from the set $S = (0, \infty)$ where $s \in S$ represent the capacity of the factory. For any given choice of s the firm's profit is drawn from a normal distribution with mean $E[x|s,\theta] = s\tilde{s} - \frac{1}{2}s^2 + \theta$ and variance $\mathrm{Var}[x|s,\theta] = s$. In words, the firm's expected profits are a concave quadratic in s with its maximum at $s = \tilde{s}$. Moreover, the risk associated with the project is proportional to the capacity of the factory.

For the purposes of this example it is assumed that $u\big(v(x)\big)$ is strictly concave in x. From proposition 2 it follows that the manager has risk-averse preferences over S. Moreover, in the absence of an incentive contract the manager's preferred strategy $\tilde{\tilde{s}}$ is strictly less than $\tilde{s}$ and thus $\mu > 0$.

The market's prior beliefs concerning the manager's talent is that θ is drawn from a normal distribution with mean $E[\theta] = \bar{\theta}$ and variance $\mathrm{Var}[\theta] = \sigma^2$. It follows that the marginal distribution is likewise normal with mean $E[x|s] = s\tilde{s} - \frac{1}{2}s^2 + \bar{\theta}$ and variance $\mathrm{Var}[x|s] = s + \sigma^2$.

The following proposition characterises the likelihood ratio for this example and identifies the range of profits that carry usable information.

Proposition 4: *For the given distribution of x and prior over θ:*

(a) *The likelihood ratio is,*

$$\frac{\hat{f}_s(x|s^*)}{\hat{f}(x|s^*)} = \frac{(x - s^*\tilde{s} + \frac{1}{2}(s^*)^2 - \bar{\theta})^2}{2(s^* + \sigma^2)^2} - \frac{(s^* - \tilde{s})(x - s^*\tilde{s} + \frac{1}{2}(s^*)^2 - \bar{\theta})}{s^* + \sigma^2} - \frac{1}{2(s^* + \sigma^2)},$$

which is convex quadratic with respect to x.

(b) $X^- = (-\infty, x^{\mathrm{strike}})$ *where* $x^{\mathrm{strike}} = \frac{1}{2}(s^*)^2 + \bar{\theta} - (\tilde{s} - s^*)\sigma^2$.

(c) $E[x|s^*] - x^{\text{strike}} = (s^* + \sigma^2)(\tilde{s} - s^*) > 0.$

Proof. Given $E[x|s]$ and $\text{Var}[x|s]$, the marginal density function is,

$$\hat{f}(x|s) = \frac{1}{\sqrt{2\pi(s+\sigma^2)}} e^{-\frac{(x - s\tilde{s} + \frac{1}{2}s^2 - \bar{\theta})^2}{2(s+\sigma^2)}},$$

while the partial derivative of $\hat{f}$ with respect to s is,

$$\hat{f}_s(x|s) = \left(\frac{(x - s\tilde{s} + \frac{1}{2}s^2 - \bar{\theta})^2}{2(s+\sigma^2)^2} - \frac{(s - \tilde{s})(x - s\tilde{s} + \frac{1}{2}s^2 - \bar{\theta})}{s+\sigma^2} - \frac{1}{2(s+\sigma^2)} \right) \hat{f}(x|s).$$

Dividing through by $\hat{f}$ and evaluating the likelihood ratio at s^* yields the likelihood ratio in (a). Taking the partial derivative with respect to x yields,

$$\frac{\partial}{\partial x}\left(\frac{\hat{f}_s(x|s^*)}{\hat{f}(x|s^*)} \right) = \frac{x - s^*\tilde{s} + \frac{1}{2}(s^*)^2 - \bar{\theta}}{(s^* + \sigma^2)^2} - \frac{s^* - \tilde{s}}{s^* + \sigma^2},$$

implying that the minimum occurs at $x^{\text{strike}} = \frac{1}{2}(s^*)^2 + \bar{\theta} - (\tilde{s} - s^*)\sigma^2$, proving (b).

A manager who selects the strategy s^* expects the profit $E[x|s^*] = s^*\tilde{s} - \frac{1}{2}(s^*)^2 + \bar{\theta}$. The amount by which the expected profit exceeds x^{strike} is,

$$E[x|s^*] - x^{\text{strike}} = (s^* + \sigma^2)(\tilde{s} - s^*) > 0,$$

where the inequality follows from the fact that $s^* < \tilde{s}$ when $\tilde{\tilde{s}} < \tilde{s}$, proving (c). □

The likelihood ratio describes the information present in the firm's profit. In this example, the likelihood ratio is convex quadratic in x which can be interpreted as follows: Extreme profits, both high and low, are indicative of risk taking as they become relatively more likely as the variance of x increases. Conversely, moderate profit levels are indicative of a risk-averse selection of s.

The second-best contract has an option structure. The manager's 'base wage' is $w(x^{\text{strike}})$. Proposition 4(b) and (12) together imply that $w(x) = w(x^{\text{strike}})$ for all $x \in X^- = (-\infty, x^{\text{strike}})$. Profits in excess of x^{strike} carry information that can be utilised by the second-best contract. It follows from (9) that $w(x)$ is strictly increasing in x for all $x \in [x^{\text{strike}}, \infty)$. Within this range the difference $w(x) - w(x^{\text{strike}})$ can be regarded the 'option component' of the manager's wage.

The location of x^{strike} is of interest here. The profit level x^{strike} is the analogue of the 'strike price' associated with stock options.[14] Proposition 4(c) states that x^{strike} is strictly less than firm's expected profit. In other words, it is possible for the manager to produce a profit that is less than market expectations and still be the beneficiary of the option component of the second-best contract. The difference $E[x|s^*] - x^{\text{strike}}$ is increasing in the variance of x, and the distance between the expected profit maximising strategy $\tilde{s}$ and the second-best strategy s^*.

This example provides a new insight into the structure of managerial incentive contracts. In industries in which there is relatively little uncertainty, and the risk of moral hazard distorting managerial decision making is relatively low, the model predicts that stock options will be included in managerial contracts, and that the strike prices will be relatively close to the current stock price. Conversely, in firms facing a high degree of uncertainty, the model predicts that the strike price will be more generous, or that managers will be vested with stock (effectively an option with a strike price of zero). This may explain why granting stock, as opposed to options, is a common practice in start-ups.

References

de Meza, D. and D. Webb (2007), Incentive Deign Under Loss Aversion, *Journal of the European Economic Association* 5, 66–92.

Dewatripont, M., Jewitt, I. and Tirole, J. (1999a), The Economics of Career Concerns, Part I: Comparing Information Structures, *Review of Economic Studies* 66, 183–198.

Dewatripont, M., Jewitt, I. and Tirole, J. (1999b), The Economics of Career Concerns,

[14] It is more accurate to say that x^{strike} is the profit outcome that gives rise to a stock price equal to the strike price.

Part II: Application to Missions and Accountability of Government Agencies, *Review of Economic Studies* 66, 199–217.

Grossman, S. and Hart, O. (1983). An Analysis of the Principal-Agent Problem, *Econometrica* 51, 7–45.

Harris, M. and Holmstrom, B. (1982), A Theory of Wage Dynamics, *Review of Economic Studies* 49, 315–333.

Holmstrom, B. (1979), Moral Hazard and Observability, *Bell Journal of Economics* 10, 74–91.

Holmstrom, B. (1999), Managerial Incentive Problems: A Dynamic Perspective, *Review of Economic Studies* 66, 169–182.

Holmstrom, B. and Ricart i Costa, J. (1986), Managerial Incentives and Capital Management, *Quarterly Journal of Economics* 101, 835–860.

Jewitt, I. (1988), Justifying the First-Order Approach to Principal-Agent Problems. Econometrica 56, 1177–1190.

Jewitt, I., O. Kadan and J. Swinkels (2008), Moral Hazard with Bounded Payments, *Journal of Economic Theory* 143, 59–82.

Mirrlees, J. (1976), The Optimal Structure of Incentives and Authority within an Organization, *Bell Journal of Economics* 7, 105–131.

Mirrlees, J. (1999), The Theory of Moral Hazard and Unobservable Behavior: Part I, *Review of Economic Studies* 66, 3–21.

Milgrom, P. (1981), Good News and Bad News: Representation Theorems and Applications, *Bell Journal of Economics* 12, 380–391.

Rogerson, W. (1985), The First-Order Approach to Principal-Agent Problems. Econometrica 53, 1357–1367.

Rosen, S. (1981), The Economics of Superstars, *American Economic Review* 71, 845–58.